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# LASER-INDUCED THERMOCAPILLARY DEFORMATION OF A THIN LIQUID LAYER

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*An equation that describes the profile of laser-induced thermocapillary deformation of the thin layer of an absorbing liquid is obtained. Numerical estimates of the deformation profile are in agreement with experimental data. It is shown that with increase in the radiation power, the deformation depth of the liquid layer increases and leads to its rupture.*

**Introduction.** The study of the thermocapillary deformation of the free surface of a liquid layer was begun by Benard [1]. In his studies, the deformation of the layer surface with a steady-state hexagonal picture of convection was noticed for the first time and attempts were made to measure the magnitude of deflection at the center of cells. According to Benard's estimates, in cells of diameter $D = 3$ mm, the deflection reaches $1.7\mu$m for a temperature difference between the center and the cell walls equal to 1 K.

In 1939, Hershey observed the formation of a thermocapillary hump in a thin water layer on a glass plate which was touched from below by a glass rod cooled in liquid nitrogen. Under a number of simplifying assumptions, he solved the Navier–Stokes equation and approximately estimated the hump height: $\Delta h = 3\Delta\sigma/(\rho g h)$ [2].

The practical need to estimate the depth of thermocapillary deformation has arisen in the varnish-and-paint industry in connection with the development of recipes of a number of decorative coatings [3]. The discovery of photoinduced capillary convection by one of the authors of the present work [4–6] and the development of several liquid-layer systems of information recording [7, 8] and the methods of determining the physical properties of thin liquid layers [9] have quickened interest to this problem [5, 10–14].

In the present paper, the stationary profile of laser-induced thermocapillary deformation is estimated and the estimates are compared with unpublished experimental results [5].

**Mathematical Model.** A Gaussian laser beam of power $P$ and diameter $2w$ is incident on the layer of an absorbing liquid of thickness $h_0$ with a characteristic horizontal size $L$ ($L/h_0 > 100$). The vertical coordinate $z$ of the thermocapillary-deflection surfaces is reckoned from the substrate: $z = 0$ (Fig. 1).

The steady flows in which the forces of inertia are small everywhere compared to the capillary pressure and the viscous friction forces, (i.e., the Reynolds number $\mathrm{Re} \ll 1$), are described by the Navier–Stokes equation $\nabla p = \mu\Delta V$; by virtue of the axial symmetry of the problem, it is convenient to write this equation in the cylindrical coordinate system $(r, \varphi, z)$ by setting $\partial/\partial\varphi = 0$ and $V_\varphi = 0$. Then,

$$\frac{1}{\rho}\,\frac{\partial p}{\partial r} = \nu\Big(\frac{\partial^2 V_r}{\partial r^2} + \frac{1}{r}\,\frac{\partial V_r}{\partial r} + \frac{\partial^2 V_r}{\partial z^2}\Big); \tag{1}$$

$$\frac{1}{\rho}\,\frac{\partial p}{\partial z} = \nu\Big(\frac{\partial^2 V_z}{\partial r^2} + \frac{1}{r}\,\frac{\partial V_z}{\partial r} + \frac{\partial^2 V_z}{\partial z^2}\Big). \tag{2}$$

An analysis of the experimentally obtained surface profile in the stationary thermocapillary convection shows that the average slope of the free surface from the deflection center to the periphery is approximately equal to 0.6. Therefore, $V_z/V_r < 1$, and Eq. (2) can be excluded from analysis in a rough approximation. Since $\partial V_r/\partial r \approx V_r/L$ and $\partial V_r/\partial z \approx V_r/h_0$, the radial velocity component in the $z$ direction changes more rapidly that in the $r$ direction. Thus, $\partial V_r/\partial r \ll \partial V_r/\partial z$. As a result, we have the equation

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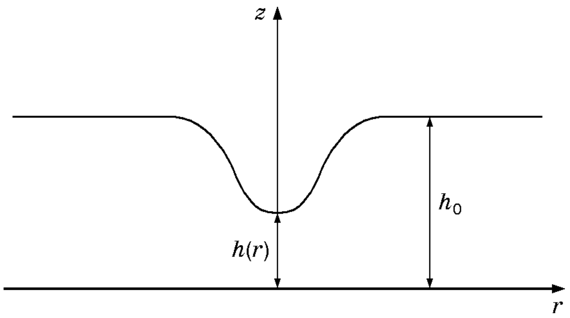

Fig. 1. Geometry of the problem.

$$\frac{\partial p}{\partial r} = \mu \frac{\partial^2 V_r}{\partial z^2}. \tag{3}$$

For a liquid layer on a rigid surface, the boundary condition

$$V_r \Big|_{z=0} = 0 \tag{4}$$

exists. The condition of equality of the viscous stress and the surface force

$$\mu \frac{\partial V_r}{\partial z}\Big|_{z=h} = \frac{\partial \sigma}{\partial r} \tag{5}$$

is satisfied on the free surface. Here $\partial\sigma/\partial r = \sigma'_T \nabla T$, where $\sigma'_T$ is the temperature coefficient of surface tension of a liquid whose value lies in the interval 0.08–0.15 mN/(m · K) for the majority of organic substances and the temperature gradient $\nabla T$ is unknown.

To integrate Eq. (3), we obtain an expression for the pressure in the liquid. Taking into account that the capillary constant $a = \sqrt{2\sigma/(g\rho)}$ is comparable with the sizes of a light spot, one can ignore the hydrostatic pressure. Then,

$$p = -\sigma\Big(\frac{1}{r}\frac{\partial h}{\partial r} + \frac{\partial^2 h}{\partial r^2}\Big). \tag{6}$$

Integrating (3) together with (4) and (5), we obtain

$$V_r = \frac{\partial p}{\partial r}\frac{z^2}{\mu} + \frac{\partial \sigma}{\partial r}\frac{z}{\mu} - \frac{\partial p}{\partial r}\frac{zh}{\mu}. \tag{7}$$

Substituting (7) into the continuity equation $\int\limits_0^h \int\limits_0^{2\pi} r V_r \, d\varphi \, dz = 0$, we obtain

$$\frac{\partial p}{\partial r} = \frac{3}{2h}\frac{\partial \sigma}{\partial r}. \tag{8}$$

From (6) and (8), we have

$$\sigma \frac{d^3 h}{dr^3} + \frac{\partial \sigma}{\partial r}\Big(\frac{3}{2h} + \frac{d^2 h}{dr^2} + \frac{1}{r}\frac{dh}{dr}\Big) + \frac{\sigma}{r}\frac{d^2 h}{dr^2} - \frac{\sigma}{r^2}\frac{dh}{dr} = 0. \tag{9}$$

Equation (9) describes the free-surface profile and it is solved numerically.

To determine $\nabla T$, generally speaking, the solution of the thermal problem is necessary, but, for simplicity, we confine ourselves to the assumption that the temperature field has the Gaussian distribution [13, 14]

$$T = T_0 + T_{\text{ch}} \exp(-r^2/w^2), \tag{10}$$

where $T_0$ is the ambient temperature and $T_{\text{ch}} = P/(4\pi k h_0)$ is the characteristic temperature ($k$ is the thermal conduction of the liquid).

As $r \to \infty$, Eq. (10) is reduced to the expression $T = T_0$, which gives the real picture of the temperature field.

For an ethyl alcohol layer [$k = 0.17$ W/(m · K)] $h_0 = 1.4 \cdot 10^{-3}$ m thick, which is irradiated by a laser of power $P \approx 8$ mW (the beam radius $w = 1.5 \cdot 10^{-3}$ m), the quantity $\nabla T$ is estimated to be 0.04 K/mm, which gives the temperature difference $\Delta T \approx 0.2$ K for a convective vortex of radius 4 mm.

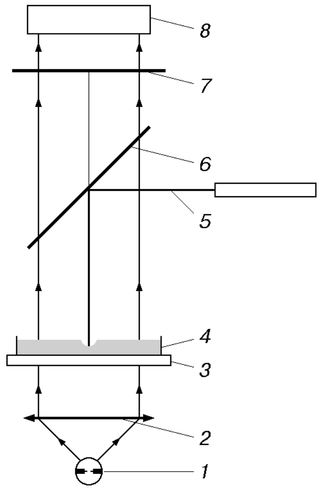

Fig. 2. Optic scheme of the photoabsorption method of determining the deformation profile: 1) photoflash; 2) collector; 3) etched glass; 4) liquid layer; 5) laser beam; 6) resonance mirror; 7) light filter; 8) photographic lens.

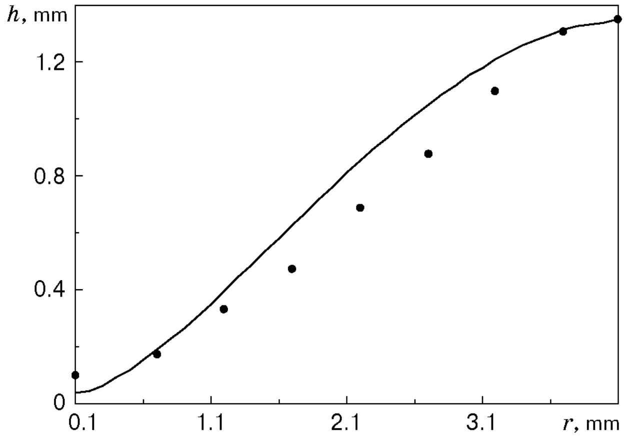

Fig. 3

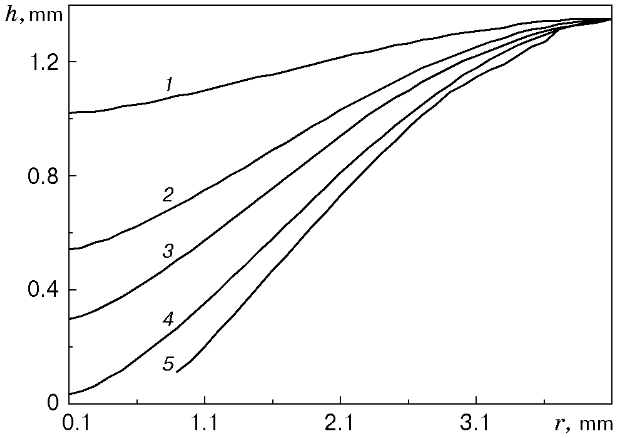

Fig. 4

Fig. 3. Calculated (solid curve) and experimental (points) profiles of the thermocapillary depression.

Fig. 4. Thermocapillary-depression profile versus the laser power for $P = 1$ (1), 4 (2), 6 (3), 8 (4), and 9 mW (5).

Equation (9) was solved by the fourth-order Runge–Kutta method with the use of the Maple V R4 software [15].

**Description of the Experiment.** To determine the profile of a depression caused by thermocapillary convection upon laser-induced local heating of a liquid layer, the photoabsorption method [5] (Fig. 2) based on the variation in the emissivity density of the negative of the layer was employed. The important element of the scheme is the resonance mirror 6, which completely reflects the laser beam 5 on the liquid layer 4 at the angle $\pi/4$ ($P \approx 8$ mW and $\lambda = 633$ nm). At the site of formation of a depression, the layer is thinner and its optic density is smaller as well. On the photograph of this part of the layer, the depression profile is connected with the profile of the emissivity densities of the negative. To find the dependence of the emissivity density and the thickness, the optic wedge formed by the same absorbing liquid was used. The wedge and the depression near which it was placed were photographed on one frame. To ensure uniform illumination, the etched glass 3 was used; this glass was illuminated by a parallel beam generated by the photoflash 1 located at the focal point of the collector 2. The application of a selective dye with the absorption peak at the wavelength $\lambda = 633$ nm in this scheme does not allow one to obtain reliable results. This is connected with the specific feature of the resonance mirror: upon complete reflection at the wavelength $\lambda = 633$ nm, it transmits almost completely the emission of an additional color, which leads to the disappearance of the contrast. This shortcoming can be eliminated if one uses dyes with two different maxima (for example, crystalline violet) or a mixture of two dyes with acute absorption maxima. Both variants were verified. In the latter case, a 3% solution of a mixture (1 : 1) of a diamond green dye and rhodamine G in a 96% ethanol was used. To increase the contrast, the green light filter 7 was placed in front of the photographic lens 8. The density profile of the negative was measured by means of an IFO-451 microdensitometer.

**Results and Discussion.** Figure 3 shows calculated (solid curve) and experimental (points) profiles of the thermocapillary depression formed in the ethyl alcohol layer of thickness $h_0 = 1.4$ mm by a laser beam of power 8 mW. Despite a number of rough simplifying assumptions, the calculated profile is in satisfactory agreement with the experimental profile, which shows the correspondence of the theoretical model to the real physical phenomenon.

Figure 4 shows calculated profiles of the thermocapillary depression depending on the laser power. With an increase in the beam power from 1 to 8 mW, the depression depth at its center increased by almost a factor of 8. The further increase in the beam power (higher than 10 mW) leads, probably, to rupture of the liquid layer, which was observed in the layers of colored benzyl alcohol whose thickness was approximately 200 $\mu$m.

**Conclusions.** An expression that determines the profile of a laser-induced stationary thermocapillary depression for $\mathrm{Re} \ll 1$ has been obtained. The numerical estimates of a depression profile formed by a laser beam in a 3% solution of a mixture (1 : 1) of a diamond green dye and rhodamine G in 96% ethanol agree satisfactorily with experimental data. In addition, the dependence of the depression profile on the laser power has been studied.

## REFERENCES

1. H. Benard, “Les tourbillons cellulaires dans une nappe liquide transportant de la chaleur par convection en regime permanent,” *Ann. Chim. Phys., Ser. 7*, **23**, 62–144 (1901).
2. A. V. Hershey, “Ridges in a liquid surface due to the temperature dependence of surface tension,” *Phys. Rev.*, **56**, 204 (1939).
3. A. D. Yakovlev, *Chemistry and Technology of Varnish-and-Paint Coatings* [in Russian], Khimiya, Leningrad (1981).
4. B. A. Bezuglyi, E. A. Galashin, and G. Ya. Dudkin, “Iodine photocondensation,” *Pis'ma Zh. Éksp. Teor. Fiz.*, **22**, No. 2, 76–79 (1975).
5. B. A. Bezuglyi, “Capillary convection controlled by the thermal action of light and its application to the methods of information recording,” Candidate’s Dissertation in Phys.-Math. Sci., Moscow (1983).
6. B. A. Bezuglyi, “Photoinduced solutocapillary convection: New capillary phenomenon,” in: *Proc. of the 1st Int. Symp. on Hydromech. Heat/Mass Transfer Microgravity* (Perm’–Moscow, 6–14 July 1991), Gordon and Breach Sci. Publ., Amsterdam (1992), pp. 335–340.
7. B. A. Bezuglyi and E. A. Galashin, “Photorensometry — new method of obtaining images,” *Zh. Nauch. Prikl. Foto Kinemat.*, **27**, No. 1, 69–71 (1982).
8. J. C. Loulergue and Xu Sen Lu, “IR photography in liquid films by thermocapillary convection,” *Int. J. Infrared Millimiter Waves*, **7**, No. 1, 171–182 (1986).
9. B. A. Bezuglyi, S. N. Lanin, and V. V. Nizovtsev, USSR Inventor’s Certificate No. 1188588, “Method of viscosity determination,” No. 3745533; Appl. 05.28.84, Publ. 10.30.85; Bull. No. 40.
10. S. Slavchev, “Photoabsorption TC-convection in the thin layer of a viscous liquid. 1. Light field,” *Teor. Prikl. Mekh.*, **18**, No. 3, 55–63 (1982).
11. A. F. Pshenichnikov and G. A. Tokmenina, “Deformation of a free liquid surface by thermocapillary motion,” *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 3, 150–153 (1983).
12. A. E. Indeikina, Yu. S. Ryazantsev, and V. M. Shevtsova, “Nonstationary thermocapillary convection in the layer of a nonuniformly heated liquid,” *Izv. Akad. Nauk SSSR, Mekh. Zhidk. Gaza*, No. 3, 17–25 (1991).
13. S. M. Pimputkar and S. Ostrach, “Transient thermocapillary flow in thin liquid layers,” *Phys. Fluids*, **23**, 1281–1285 (1980).
14. H. Helmers and W. Witte, “Holographic study of laser-induced liquid surface deformations,” *Opt. Comm.*, **49**, 21–23 (1984).
15. V. P. Dyakonov, *Maple V R3/R4/R5 Mathematical System* [in Russian], Solon, Moscow (1998).